Quantum Cloning for Absolute Radiometry

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In the quantum regime information can be copied with only a finite fidelity. This fidelity gradually increases to 1 as the system becomes classical. In this Letter we show how this fact can be used to directly measure the amount of radiated power. We demonstrate how these principles can be used to build a practical primary standard.

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Since its inception quantum mechanics has had a deep tie with radiometry, the science of measurement of electromagnetic radiation. The electrical substitution radiometer, developed by Lummer and Kurlbaum in 1892 [1], was used to observe the spectral distribution of a heated blackbody. In 1900 Planck was able to describe this distribution by assuming that electromagnetic radiation could only be emitted in multiples of an energy quantum $E = h\nu$. This discovery not only provided an accurate law relating the radiated spectral density to temperature, but laid the foundations of quantum physics. The electrical substitution radiometer is still used as the primary standard for spectral radiance by many metrology laboratories. These systems have been improved over more than a century and can now achieve absolute uncertainties better than 10^{-4}, when operated at relatively high powers [2].

More recently, nonlinear optical effects such as spontaneous parametric down-conversion have provided a new primary standard based on the correlations of quantum fields [3]. The accuracy of these techniques has improved by nearly 1 order of magnitude every 10 years, and is currently of the order of 10^{-3}. These systems are currently limited to the photon-counting regime, with a recent theoretical proposal for extension to higher photon rates [4].

In this Letter, we present a radiometer that overcomes these limitations and works over a broad range of powers: from the single photon level up to several tens of nW ($\approx 10^{11}$ photons/s), i.e., from the quantum to the classical regime. In fact, our system is able to provide an absolute measure of spectral radiance by relaying on a particular aspect of the quantum to classical transition: as the number of information carriers (photons) grows, so does the fidelity with which they can be cloned. For an optimal cloning machine [5–9] this relation can be derived *ab initio* [10,11] so that a measurement of the fidelity of the cloning process is equivalent, as we shall see below, to an absolute measurement of spectral radiance.

Optimal cloning has been demonstrated in a variety of systems [6,8,9]. Stimulated emission in atomic systems is particularly practical as high gains can be easily achieved and the entire system can be implemented in fiber, which both ensures the presence of a single spatial mode and makes the system readily applicable, though not limited, to current telecommunications technology.

Principle of operation.—The aim of this experiment is to produce an absolute measurement of luminous power P_{in}. We will do this by using an optimal universal quantum cloning machine (QCM). As we shall see, such a device is able to directly relate a relative measurement of two orthogonal polarizations at the device's output to P_{in}. The relative measure that we use is the fidelity $\mathcal{F}$, which is the mean overlap between the input and output polarization and can be expressed as follows:

$$\mathcal{F} = \frac{P_{\parallel}}{P_{\parallel} + P_{\perp}}, \tag{1}$$

where $P_{\parallel}$ and $P_{\perp}$ are the output powers in the polarizations parallel and perpendicular to the polarization of the input light.

For an optimal QCM, the fidelity of a cloning process from N to M qubits can be derived *ab initio* [10] to be

$$\mathcal{F}_{N\to M} = \frac{NM + N + M}{NM + 2M}. \tag{2}$$

This equation remains valid when we clone a large number of polarization qubits distributed over a large number of temporal modes and can be rewritten in terms of the average number of input and output photons per (temporal) mode μ_{in} and μ_{out} [8]:

$$\mathcal{F}_{\mu_{\mathrm{in}}\to\mu_{\mathrm{out}}} \equiv \frac{\mu_{\parallel}}{\mu_{\parallel} + \mu_{\perp}} = \frac{\mu_{\mathrm{in}}\mu_{\mathrm{out}} + \mu_{\mathrm{in}} + \mu_{\mathrm{out}}}{\mu_{\mathrm{in}}\mu_{\mathrm{out}} + 2\mu_{\mathrm{out}}}, \tag{3}$$

where μ_{out} contains both a number of exact copies of the input signal and intrinsic noise due to the amplification process, i.e., $\mu_{\mathrm{out}} = \mu_{\parallel} + \mu_{\perp}$.

It is also possible to express μ_{out} as a function of μ_{in} and the amplifier gain G [12]: μ_{out} is the sum of the stimulated emission $G\mu_{\mathrm{in}}$ and the spontaneous emission, equivalent to amplifying the vacuum, so that

$$\mu_{\mathrm{out}} = G\mu_{\mathrm{in}} + 2(G - 1). \tag{4}$$

Equations (3) and (4) can be combined to obtain the spectral radiance μ_{in} as a function of fidelity and gain,

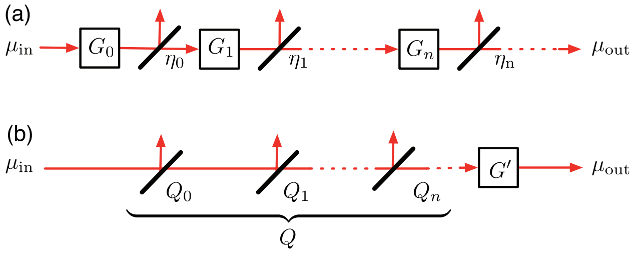

FIG. 1 (color online). (a) Model of a nontotally inverted medium as succession of infinitesimal gain elements G_n and loss elements η_n. (b) Each loss element η_n within the fiber is equivalent to a *smaller* loss element Q_n before the amplifier.

$$\mu_{\rm in} = \frac{2\mathcal{F}G - G - 2\mathcal{F} + 1}{G - \mathcal{F}G} \simeq \frac{2\mathcal{F} - 1}{1 - \mathcal{F}}, \quad (5)$$

with the approximation holding for $G \gg 1$. $P_{\rm in}$ can be derived from $\mu_{\rm in}$ and a measurement of the number of modes per unit time τ_c^{-1}.

Three aspects make this scheme attractive. The first is that after amplification input power information is polarization encoded and is therefore insensitive to losses [13]. The second is that the experiment can be performed entirely in fiber, ensuring the selection of a single spatial mode. The third advantage is that this scheme works over a broad scale of powers, from single photon levels up to several tens of nW ($\sim 10^{11}$ photons/s).

Nonideal cloning.—The reasoning presented above assumes the universal cloning process to be optimal. It has been shown theoretically that amplification in an inverted atomic medium indeed provides optimal cloning [5]. However, for precision applications it is important to consider the possible effects of a nonperfectly inverted medium, which we model by a succession of infinitesimal gain and loss elements, G_n and η_n, as shown in Fig. 1(a). Note that the value associated with a loss element is its transmittance so that $\eta_n = 1$ for perfect transmission (no losses) and $\eta_n = 0$ for zero transmission. We have shown [14] that this model is equivalent to a much simpler one. Each loss element η_n can be represented by a different loss element Q_n before an optimal cloning machine with gain G', as shown in Fig. 1(b). It can be shown that the product Q of all Q_n can be expressed as

$$Q = \prod Q_n, \qquad Q_n = \frac{G_0^n \eta_n}{G_0^n \eta_n + (1 - \eta_n)}, \quad (6)$$

where G_0^n is the effective gain between the beginning of the amplifier and element η_n. A fully inverted medium would have $Q = 1$. From Eq. (6) it is apparent that the effect of a small loss ($\eta_n \lesssim 1$) is proportional to $1/G_0^n$. As G_0^n grows exponentially over the length of the fiber, losses towards the end can be neglected. At the beginning of the amplifier two effects guarantee that the medium is fully inverted: the input signal is small, as it has not been amplified yet, and the signal and pump copropagate, ensuring maximum pump power in this region. Cloning optimality can then be achieved in a nonideal amplifier.

Experimental arrangement.—Figure 2 shows the setup, which can be conceptually divided in three main parts: generation of a set amount of power, amplification, and fidelity measurement. To test our system, we prepare states with a known number of photons per mode ($\mu_{\rm in}$). This is done using a polarized light-emitting diode (LED) that is passed through a polarization scrambler and a variable attenuator. The scrambler chooses a new random polarization before each experiment, which is repeated at a rate of 200 Hz.

The power is then split (50:50), with one branch monitored on a calibrated power meter, while the other is sent to the amplification stage. Amplification is provided by 2 m of Er^{3+} doped fiber (attenuation 16.7 dB/m at 1530 nm), pumped by a 980 nm diode laser. The pump light is combined with the signal on the input of the Er^{3+} fiber using a wavelength division multiplexer (WDM), and an isolator is placed before the input to prevent unwanted resonances. After the Er^{3+} doped fiber, most of the pump power is removed using an additional WDM. In this realization, the no-cloning theorem is guaranteed by the Er^{3+} spontaneous emission, which adds randomly polarized photons to the signal. We used an optical frequency-domain reflectometer [15] to verify that the gain per unit length is constant over the entire fiber, indicating that the atomic medium is fully inverted. Results are shown in Fig. 3. The measurement stage consists of a grating-based tunable filter and a polarimeter. The filter has a width of 273.3(5) pm (FWHM), which ensures that the effects of polarization mode dispersion can be neglected. The polarimeter measures the degree of polarization (DOP) with a

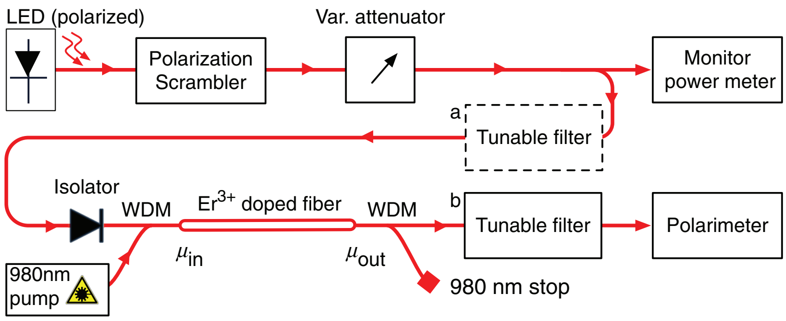

FIG. 2 (color online). Experimental arrangement: a broadband source with controllable polarization and power is amplified by an erbium doped fiber amplifier. The degree of polarization (DOP) of the amplified light is then measured with a polarimeter. Spectral bandwidth is determined by a tunable filter. A value for the input power can be calculated from the DOP and compared with a calibrated power meter (monitor).

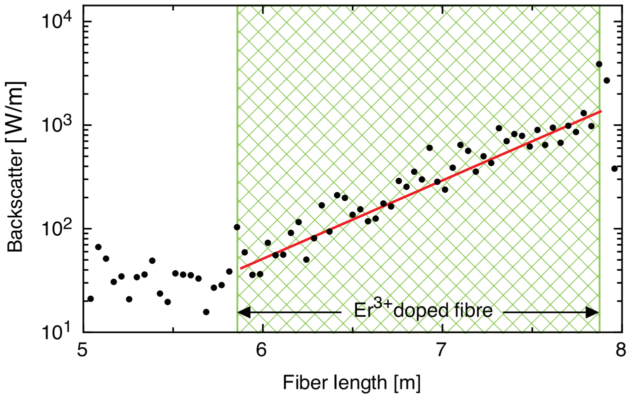

FIG. 3 (color online). Optical frequency-domain reflectometer measurement showing homogenous gain per unit length within the Er^{3+} doped fiber. The solid line is an exponential fit of the data.

nominal accuracy of 1%, where the DOP is defined as the polarized power (in any basis) $P_{\rm pol}$ over the total power $P_{\rm tot}$, and is related to fidelity by $\mathcal{F} = (1 + \mathrm{DOP})/2$. Using a polarimeter rather than simply a polarizing beam splitter and power meter is less accurate but allows us to test whether the system works equally well for arbitrary input states of polarization, i.e., whether the QCM is truly universal.

Experimental procedure.—To evaluate the accuracy of our system, we will compare our measurements of $\mu_{\rm in}$, obtained using the cloning radiometer, with those obtained with the reference (monitor) power meter and denoted by a star, i.e., $\mu^*_{\rm in}$. To obtain $\mu^*_{\rm in}$ from the reference power meter we first measure the ratio between the power at the monitor output and the power at the entrance of the amplifier within the bandwidth of the tunable filter. This is done by placing the filter just before the amplification stage (position "*a*" in Fig. 2). Together with a measurement of the filter's attenuation and bandwidth, this allows us to obtain $\mu^*_{\rm in}$ from the monitor power. The filter is then placed after the amplification stage (position "*b*") so that Er^{3+} spontaneous emission outside the bandwidth of interest is eliminated. We then vary $\mu^*_{\rm in}$ using the attenuator and record the monitor power versus the degree of polarization. For each $\mu^*_{\rm in}$ the measurements are repeated for 20 different input polarizations to estimate uncertainties.

Results.—The fidelity of the cloning process is a measure of spectral radiance. In order to measure power it is necessary to have an accurate measure of the number of modes involved. Using a single-mode fiber ensures that there is only a single spatial mode: only the number of temporal modes per second need to be measured. It is convenient to define the coherence time τ_c as in [16]

$$\tau_c = \int_{-\infty}^{\infty} |\gamma(\tau)|^2 d\tau, \tag{7}$$

where $\gamma(\tau)$ is the autocorrelation function normalized such that $\gamma(0) = 1$. Using this definition, the coherence length $c\tau_c$ is the length of the unit cell of photon phase space [16], so that the number of modes per second is simply τ_c^{-1}. Measuring this value with an optical low-coherence interferometer (Fig. 4) yields $\tau_c = 19.71(4)$ ps which corresponds, assuming a Gaussian shape, to wavelength FWHM of $\Delta\lambda = 273.3(5)$ pm. We also performed a (less precise) spectrometric measurement yielding $\Delta\lambda = 271$ pm. With this filter, a mean of one photon per temporal mode corresponds to 6.461 nW. We measure the amplifier gain to be $G = 66.5(3)$ by directly comparing the power at the output of the amplifier with the power at the input. The inset of Fig. 5 shows a typical plot, in terms of $\mu^*_{\rm in}$ and $\mu^*_{\rm out}$, where $\mu^*_{\rm out}$ is the value of $\mu_{\rm out}$ obtained from the polarimeter's internal power meter. The thickness of the line represents random errors. Note that the gain is $G = \partial\mu_{\rm out}/\partial\mu_{\rm in}$, so that any systematic error in either the power measurement or the estimation of the number of modes cancels. The line in the inset of Fig. 5 is a fit of the data for $\mu^*_{\rm in} < 1$, revealing that at high $\mu_{\rm in}$ the gain is reduced. This effect could be minimized by pumping from both sides of the Er^{3+} doped fiber. Nevertheless, the gain is constant for $\mu_{\rm in} < 2$, allowing us to assume within this range that the intercept μ_0 corresponds to the spontaneous emission $(2G - 2)$ from Eq. (4), so that $\mu_{\rm out} = G\mu_{\rm in} + \mu_0$. In this range it is then possible to measure $\mu_{\rm in}$ without distinguishing the polarizations, as $\mu_{\rm in} = 2(\mu^*_{\rm out} - \mu^*_{\rm in})/\mu^*_0 - 2$.

We then measure the fidelity $\mathcal{F}$ versus $\mu^*_{\rm in}$: Fig. 5 shows a typical plot, which can be fitted with Eq. (5), where $\mu_{\rm in}$ has been replaced with $k\mu^*_{\rm in}$ and k is the fitted parameter. With this definition $k = \mu_{\rm in}/\mu^*_{\rm in}$ represents the discrepancy between our measurement of $\mu_{\rm in}$ and the value $\mu^*_{\rm in}$ obtained from the reference power meter. Here, k also accounts for the possibility of nonoptimal cloning which would introduce a further factor $Q \leq 1$, equivalent to a loss at the input of the cloning machine. The fitted curve in Fig. 5 yields $k = 1.013(5)$, where the error indicated represents statistical uncertainty.

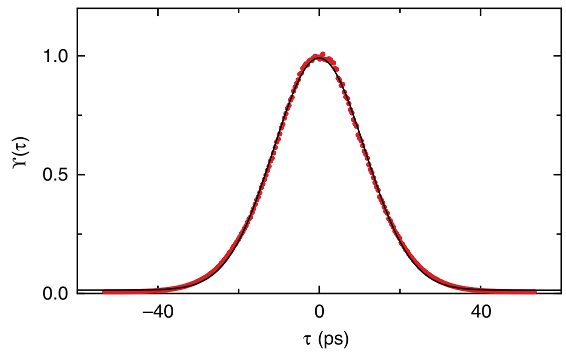

FIG. 4 (color online). Autocorrelation function of the source after the filter. $\gamma(\tau)$ is the fringe visibility measured with a low-coherence interferometer; τ_c will simply be the numerical integral of the square of this data. The solid line is a Gaussian fit.

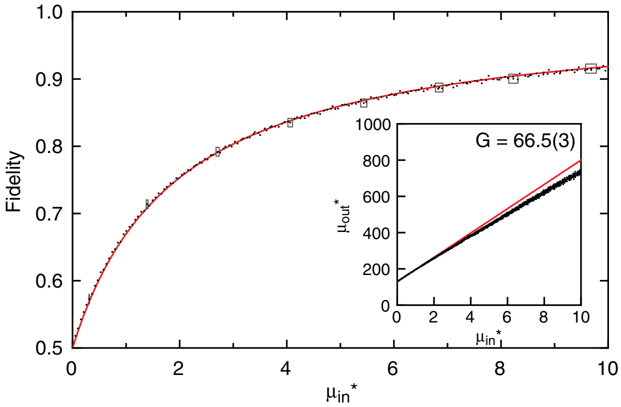

FIG. 5 (color online). Fidelity versus number of input photons per mode, fitted with Eq. (5). Representative errors are shown as boxes on some points. The inset shows the output versus input number of photons per mode, where the line is a fit on the first data points ($\mu_{in}^* < 1$), showing reduced gain as μ_{in} grows.

Error estimation [14].—One of the advantages of this technique is that relative measurements, which usually have small errors, are used, but how does a small uncertainty in the fidelity $\Delta\mathcal{F}$ translate into an error in the measurement of μ_{in}? From Eq. (5), assuming $G \gg 1$ we obtain

$$\Delta\mu_{in} = (2 + \mu_{in})^2 \Delta\mathcal{F}. \tag{8}$$

$\Delta\mu_{in}/\mu_{in}$ has a minimum of $\Delta\mu_{in}/\mu_{in} = 8\Delta\mathcal{F}$ at $\mu_{in} = 2$, i.e., when spontaneous and stimulated emissions are equal. At higher spectral radiances, $\Delta\mu_{in}/\mu_{in}$ rises linearly with μ_{in}. The spectral bandwidth of the filter can be chosen as to operate in the desired power regime: our system is optimal at 13 nW. Commercially available filters would allow this point to be easily lowered to 100 pW. From preliminary tests we estimate that this technique would work to an upper limit of 100 nW, above which the effects of polarization mode dispersion and wavelength dependence of the components need to be taken into account.

The two main systematic uncertainties in our system are due to the reference power meter, and to the polarization measurement. The power meter is an EXFO PM-1100, recently calibrated by METAS to an absolute uncertainty of 0.7% and with a measurement to measurement standard deviation of 0.5% (including fiber reconnection). The linearity of this power meter is within this uncertainty over its entire range. The fidelity is measured with a Profile PAT 9000 polarimeter which has a nominal $\Delta\mathcal{F} = 5 \times 10^{-3}$. Systematic error is dominated by the polarimeter, so that $\Delta\mu_{in}/\mu_{in} \sim 4\%$ for $\mu_{in} = 2$.

Conclusion.—We have shown that the fidelity of cloning can be used to produce an absolute power measurement with an uncertainty only limited by the uncertainty of a relative power measurement. We demonstrate the scheme with an all-fiber experiment at telecommunications wavelengths, achieving an accuracy of 4%, not fundamentally limited by the technique but rather by the usual complications that arise when building a primary standard, leaving room for improvement by a metrology laboratory. The experiment also demonstrates optimal $1 \rightarrow 67$ cloning and is an interesting application of quantum information science and of the study of the quantum to classical transition.

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