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Abstract. Bell's theorem is a no-go theorem stating that quantum mechanics
cannot be reproduced by a physical theory based on realism, freedom to choose
experimental settings and two locality conditions: setting (SI) and outcome (OI)
independence. We provide a novel analysis of what it takes to violate Bell's
inequality within the framework in which both realism and freedom of choice
are assumed, by showing that it is impossible to model a violation without
having information in one laboratory about *both* the setting and the outcome
at the distant one. While it is possible that outcome information can be revealed
from shared hidden variables, the assumed experimenter's freedom to choose the
settings ensures that the setting information *must be* non-locally transferred even
when the SI condition is obeyed. The amount of transmitted information about
the setting that is sufficient to violate the CHSH inequality up to its quantum
mechanical maximum is 0.736 bits.

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Bell's inequalities are certain constraints on correlations between space-like separated measurements that are satisfied in any local realistic theory [1]. The inequalities are violated by quantum predictions for some entangled states. The usual set of assumptions invoked in the derivation of Bell's inequalities comprises *realism*, the experimenter's *freedom* to choose the measurement settings ('freedom of choice') and two *locality* conditions: setting independence (SI) and outcome independence (OI) [2]–[4]. Maintaining realism and freedom of choice thus necessitates an exchange of information between distant measurement stations that defies locality so as to violate one (or both) of the independence conditions. What kind and amount of information has to be transferred between the stations to model the violation of Bell's inequalities? Is it information about the distant outcome, or about the distant setting, or about both? These questions will be addressed in the present paper from an information theoretical perspective, thereby providing a novel analysis of what it takes to violate Bell's inequality.

While we see other possibilities than maintaining realism and freedom of choice, and introducing non-local actions to interpret the implications of Bell's theorem, we acknowledge the importance of exploring alternative descriptions to deepen our understanding of the foundations of quantum theory. In addition to fundamental reasons, answering the above questions is important in quantum information, such as, for example, in quantum communication complexity problems [5]. The question of how much setting and/or outcome information needs to be exchanged in a Bell experiment for a given degree of violation is relevant to quantify the classical resources required to simulate quantum efficiency in these problems.

In this work, we assume realism and freedom of choice, and we study non-local hidden-variable models with one-way communication between two separated observers, conventionally called Alice and Bob. Clearly, communicating a distant setting allows the simulation of violation of Bell's inequality. For example, we could let Bob's outcome be determined by a shared hidden variable, and we could have Alice's outcome depend not only on her local setting and the shared hidden variable, but also on Bob's setting. In this way, any set of correlations could be modeled because, at Alice's location, there is information about all the outcomes and settings that are involved.

Here we show that information about a distant setting and outcome is not only sufficient to simulate violation of Bell's inequality but also necessary. This is contrasted with the well-known condition [3, 4] that either OI or SI for (hidden-variable) conditional probabilities *can still be obeyed* in models producing a violation of Bell's inequality. Note that it is not a contradiction that for a violation of Bell's inequality (I) both distant setting and outcome information must be locally available and (II) OI or SI can still be fulfilled. This is because (I) and (II) refer to different notions. For instance, OI is obeyed when the conditional (hidden variable) probability of Alice's outcome does not *explicitly* depend on Bob's outcome. However, the information about Bob's outcome can still be *implicitly* contained in the shared hidden variable. This allows for a novel analysis of what it takes to violate Bell's inequality.

Furthermore, we show that, while it is possible that information about the distant outcome can be read from the hidden variables received from the source, information about the setting *must* be non-locally transmitted, implicitly or explicitly, in any model where the experimenters are free to choose their settings. We are able to trace this asymmetry between setting and outcome information to the freedom of the experimenters to choose their settings. We furthermore apply our analysis to the non-local hidden-variable models of Toner and Bacon [6], Leggett [7] and Bohm [8]. Finally, we show that the amount of transmitted information about

the setting that is sufficient to violate the Clauser–Shimony–Holt (CHSH) inequality up to its quantum mechanical maximum is 0.736 bits.

We begin with the usual formal definitions of the assumptions of Bell's theorem. In our notation, a and b stand for the measurement settings chosen by the two distant experimenters Alice and Bob, respectively; A and B denote their respective measurement outcomes, and λ denotes a set of hidden variables.

(i) For stochastic (probabilistic) hidden-variable theories, the assumption of *realism* dictates that the hidden variable λ specifies joint (non-negative, properly normalized) probabilities $P(A_{1,1}, A_{1,2}, A_{2,1}, ..; B_{1,1}, B_{1,2}, B_{2,1}, ..|\lambda)$, where, for example, the result $A_{1,2}$ of Alice for her setting choice 1 can depend on some non-local parameter 2, typically Bob's setting choice. The conditional probabilities $P(A, B|a, b, \lambda)$ that will be used in this paper are then obtained as marginals of these joint ones.

(ii) SI, often also called parameter independence [3], is the part of the locality condition that prohibits the conditional dependence of the probability to obtain the outcome in one laboratory on the choice of the setting at the other one: $P(A|a, b, \lambda) = P(A|a, \lambda)$, and analogous for $P(B|\cdot)$. Similarly, under OI, Alice's probability of obtaining her outcome does not conditionally depend on Bob's outcome, and vice versa: $P(A|a, b, B, \lambda) = P(A|a, b, \lambda)$, again analogous to $P(B|\cdot)$. The conjunction of these two conditions is equivalent to Bell's condition of local causality [2]–[4], [9]: $P(A, B|a, b, \lambda) = P(A|a, \lambda)P(B|b, \lambda)$. The latter condition allows us to define the joint probabilities from (i) as $p(A_1, A_2|\lambda)p(B_1, B_2|\lambda)$, where, for example, A_1 is the result of Alice for her setting choice 1 [10]. Note that fulfillment of SI does *not* imply that $P(A|a, b, B, \lambda)$ equals $P(A|a, B, \lambda)$. This shows that SI does not entail *complete* independence from the distant setting and that different criteria should be used.

(iii) The experimenter's *freedom of choice* to choose the measurement setting ensures that the selected measurement setting is statistically independent of the hidden variables sent by the source (even in a deterministic model) [11, 12]. In terms of the (Shannon) mutual information, this assumption is expressed as $I(\lambda : a) = I(\lambda : b) = 0$. As we will show, the assumption of freedom of choice is responsible for the *fundamental asymmetry* between settings and outcomes because it guarantees that the settings, contrary to the outcomes, are considered as independent variables.

Under these three assumptions, the CHSH inequality [13] must be obeyed,

$$\frac{1}{4} \sum_{a,b=0}^{1} P(A \oplus B = ab|a, b) \leqslant \frac{3}{4}, \tag{1}$$

with $\oplus$ denoting addition modulo 2. We let Alice and Bob each choose with 50% probability one of two settings, $a, b = 0, 1$, and obtain measurement results, $A, B = 0, 1$, respectively. (Both sides in inequation (1) are divided by 4 for later convenience.)

Assuming freedom of choice and realism, violations of the CHSH inequality imply that either OI or SI, or both, needs to be given up. In the framework of 'experimental metaphysics' [3], it is violation of the condition OI that is supposed to be responsible for the violation of the CHSH inequality, and it is extensively argued by many philosophers of this school that this is not an instance of 'action at a distance' but only of some innocent 'passion at a distance': one passively comes to know the faraway outcome, but one cannot actively change it. In contrast, violation of SI allows superluminal signaling at the hidden variable level, because the distant observer can freely choose his measurement settings. Our analysis shows that SI and OI do not provide us with the full picture of what (non-) local information is

needed in violations of Bell inequalities. Both SI and OI are in fact conditions on the conditional statistical independence of probabilities for the *local* outcome only. They do not exhaust all the possibilities of how information about *distant* settings and outcomes can be inferred (locally or non-locally). Here we will provide such an analysis. We thus no longer focus on conditional statistical independence, but instead on the availability of (non-) local information.

We will show that within the framework of non-local realistic theories it is impossible to model a violation of the CHSH inequality without having information in one laboratory about *both* the setting and the outcome at the distant one. Thus, the availability of non-local information displayed by models that violate the CHSH inequality is necessarily about both the non-local settings and the outcomes, despite the fact that it is *not necessary* that the models are *both* explicitly setting dependent (¬ SI) *and* outcome dependent (¬ OI).

In order to prove our results, we consider a local hidden-variable model augmented by information available to Alice about Bob's laboratory. While it is not necessary, it is instructive to think about this information as one-way classical communication from Bob to Alice. In every run of the experiment, Alice and Bob first choose their settings (a and b) and receive hidden variables λ, which are independent of the choice of the settings. Then, Bob (or some process in his laboratory) generates the outcome B, which in general depends on λ and b. Next, Bob generates the *message* $\mathcal{X}$, which depends on λ, b and B. The generation of B and the generation of $\mathcal{X}$ are, in general, probabilistic processes. It is assumed that the exact mechanism of how B and $\mathcal{X}$ are generated is known to Alice. Finally, $\mathcal{X}$ is transmitted to Alice, who uses her optimal strategy, based on the knowledge of her setting a, the shared hidden variables λ, Bob's mechanisms and the message $\mathcal{X}$, to produce her outcome A in order to maximally violate the CHSH inequality.

From Alice's perspective, the CHSH inequality reads

$$\tfrac{1}{2}P(A = B|a = 0) + \tfrac{1}{2}P(A = B \oplus b|a = 1) \leqslant \tfrac{3}{4}, \tag{2}$$

where, for example, $P(A = B|a = k)$ is the probability that the outcome of Alice equals that of Bob, given that she has chosen the kth setting. We shall show that the probabilities entering inequation (2) can be interpreted as a measure of the information that Alice has about Bob's measurement setting and outcome. For this purpose, we introduce the 'guessed "information"' (GI)[8],

$$J(\mathcal{X} \to \mathcal{Y}) := \sum_i P(\mathcal{X} = i) \max_j [P(\mathcal{Y} = j|\mathcal{X} = i)], \tag{3}$$

where $\mathcal{X}$ takes values $i = 1, \ldots, X$ and $\mathcal{Y}$ takes values $j = 1, \ldots, Y$. The value of $J(\mathcal{X} \to \mathcal{Y})$ gives the average probability of correctly guessing $\mathcal{Y}$ knowing the value of $\mathcal{X}$. Its maximum is 1 and then $\mathcal{Y}$ is fully specified by $\mathcal{X}$. The minimum of $J(\mathcal{X} \to \mathcal{Y})$ equals $\frac{1}{Y}$ and then $\mathcal{X}$ reveals no information about $\mathcal{Y}$. We note that GI reaches its minimum when the mutual information is $I(\mathcal{X} : \mathcal{Y}) = 0$, and it is maximal when $I(\mathcal{X} : \mathcal{Y}) = \log Y$. As an example, freedom of choice can be stated as $J(\lambda \to a, b) = \frac{1}{4}$, i.e. λ cannot reveal any information about the settings a and b. This implies the weaker condition $J(\lambda \to b) = \frac{1}{2}$, which is sufficient for our analysis, and to which we refer as freedom of choice throughout the rest of the paper.

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[8] This measure of information is related to conditional min-entropy. Min-entropy is defined by $H_{\min}(Y) = -\log(\max P(Y))$, and the conditional one reads $H_{\min}(Y|X) = \sum_x P(x) H_{\min}(Y|x) = -\sum_x P(x) \log(\max P(Y|x))$. This is just our definition (3), with the only difference being the log function. For this reason, a case could be made to use the terminology 'guessed probability' rather than 'GI'.

Alice now uses an optimal maximization strategy so as to maximally violate the CHSH inequality. Consider the case in which Alice chooses $a = 0$. Her goal is to maximize the probability $P(A = B|a = 0)$ given the communicated value $\mathcal{X}$ and the received hidden variables λ. This maximized probability is just the average probability of correctly guessing B given $\mathcal{X}$ and λ: $J(\lambda, \mathcal{X} \to B)$. Similarly, if her setting is $a = 1$, the maximal probability $P(A = B \oplus b|a = 1)$ equals $J(\lambda, \mathcal{X} \to B \oplus b)$. This allows us to phrase the CHSH inequality in terms of the GIs,

$$\frac{1}{2}J(\lambda, \mathcal{X} \to B) + \frac{1}{2}J(\lambda, \mathcal{X} \to B \oplus b) \leqslant \frac{3}{4}. \tag{4}$$

In case Alice's scheme is not optimal, $P(A = B|a = 0)$ is upperbounded by $J(\lambda, \mathcal{X} \to B)$, and likewise $P(A = B \oplus b|a = 1)$ is upperbounded by $J(\lambda, \mathcal{X} \to B \oplus b)$. Therefore, even if Alice's strategy is not optimal, violation of (4) is necessary for a violation of the CHSH inequality. This holds for any of the eight different CHSH inequalities.

We are now in a position to prove that a *necessary* condition for the violation of Bell's inequalities within non-local realism is that information both about the setting and about the outcome produced at one laboratory must be available at the distant laboratory. If there is no outcome information available, i.e. $J(\lambda, \mathcal{X} \to B) = \frac{1}{2}$, the left-hand side of inequation (4) cannot exceed $\frac{3}{4}$. To prove that setting information is also necessary, note that if one knows both B and $B \oplus b$, one also knows b. Thus, the average probability of correctly guessing b is greater or equal to the product of the average probabilities of correctly guessing of B and $B \oplus b$,

$$J(\lambda, \mathcal{X} \to b) \geqslant J(\lambda, \mathcal{X} \to B)J(\lambda, \mathcal{X} \to B \oplus b). \tag{5}$$

If $\mathcal{X}$ and λ carry no information about the setting, i.e. $J(\lambda, \mathcal{X} \to b) = \frac{1}{2}$, inequation (5) can be rewritten as $J(\lambda, \mathcal{X} \to B \oplus b) \leqslant \frac{1}{2}J^{-1}(\lambda, \mathcal{X} \to B)$, which implies

$$\frac{1}{2}J(\lambda, \mathcal{X} \to B) + \frac{1}{2}J(\lambda, \mathcal{X} \to B \oplus b) \leqslant \frac{1}{2}J(\lambda, \mathcal{X} \to B) + \frac{1}{4J(\lambda, \mathcal{X} \to B)} \tag{6}$$

for the left-hand side of (4). This value is less than or equal to $\frac{3}{4}$ for the whole range of $J(\lambda, \mathcal{X} \to B) \in [\frac{1}{2}, 1]$. Thus, if there is no setting information, the violation of inequation (4), or inequation (1), is impossible.

Although information both about the distant setting and about the distant outcome must be available at the local laboratory to have a violation, we show that, given freedom of choice, information about the distant setting has to be transmitted non-locally, whereas it is possible that information about the distant outcome can be obtained without any transmission from the shared hidden variables. This is shown by a further analysis of what information has to be transmitted via the message $\mathcal{X}$, over and above the information in the hidden variable λ. This also allows us to analyze the above-mentioned asymmetry between the outcome information and the setting information in a more formal way.

To this end, we introduce a measure of information that we call 'transmitted "information"' (TI), which is the difference between the averaged probability of correctly guessing the value of the variable $\mathcal{Y}$ when knowing $\mathcal{X}$ and λ, and the probability when knowing only λ,

$$\Delta_\lambda(\mathcal{X} \to \mathcal{Y}) := J(\lambda, \mathcal{X} \to \mathcal{Y}) - J(\lambda \to \mathcal{Y}). \tag{7}$$

$\Delta_\lambda(\mathcal{X} \to \mathcal{Y})$ takes values between 0 and $1 - \frac{1}{Y}$. Its lowest value means that transmission of $\mathcal{X}$ does not increase Alice's chances of guessing the correct value of $\mathcal{Y}$; $\mathcal{X}$ carries no *new* information about $\mathcal{Y}$ that is not already available to Alice through λ.

We have already established that either $J(\lambda, \mathcal{X} \to B) = \frac{1}{2}$ or $J(\lambda, \mathcal{X} \to b) = \frac{1}{2}$ implies no violation of the CHSH inequality. The asymmetry between the outcome and setting information originates from the freedom of choice assumption $J(\lambda \to b) = \frac{1}{2}$, which leads to

$$J(\lambda, \mathcal{X} \to b) = \Delta_\lambda(\mathcal{X} \to b) + \frac{1}{2}. \tag{8}$$

We see that $\Delta_\lambda(\mathcal{X} \to b) = 0$ leads to $J(\lambda, \mathcal{X} \to b) = \frac{1}{2}$, which means no violation of the CHSH inequality. On the other hand, there is no assumption corresponding to freedom of choice regarding the outcomes, i.e. there are no physical grounds for assuming $J(\lambda \to B) = \frac{1}{2}$. Instead, one has

$$J(\lambda, \mathcal{X} \to B) = \Delta_\lambda(\mathcal{X} \to B) + J(\lambda \to B). \tag{9}$$

Thus, even if $\Delta_\lambda(\mathcal{X} \to B) = 0$, it is possible that $J(\lambda, \mathcal{X} \to B) > \frac{1}{2}$, if $J(\lambda \to B) > \frac{1}{2}$. Also, $J(\lambda \to B) = \frac{1}{2}$ does not mean that $J(\lambda, \mathcal{X} \to B) = \frac{1}{2}$ since $\Delta_\lambda(\mathcal{X} \to B)$ can be greater than 0. Summing up, neither $\Delta_\lambda(\mathcal{X} \to B) = 0$ nor $J(\lambda \to B) = \frac{1}{2}$ individually implies no violation of the CHSH inequality, although both of them together do. One can easily construct a toy model[9] where $J(\lambda \to B) = \frac{1}{2}$ and violation occurs because the TI is $\Delta_\lambda(\mathcal{X} \to B) = \frac{1}{2}$. A toy model where $\Delta_\lambda(\mathcal{X} \to B) = 0$ and violation occurs is presented later. All the different cases are presented in table 1, where our technical results are also contrasted with those in terms of the conditions OI and SI.

To reinforce our conclusion that the freedom of choice assumption is responsible for the asymmetry, consider the possibility of 'superdeterminism', where everything is determined by the hidden variables, even the setting choices. In that case, both $J(\lambda \to b) = 1$ and $J(\lambda \to B) = 1$, and consequently we have both $\Delta_\lambda(\mathcal{X} \to b) = 0$ and $\Delta_\lambda(\mathcal{X} \to B) = 0$. In that case there simply is no new information to be transferred, i.e. $\mathcal{X}$ is redundant as λ determines all there is to know. Settings and outcomes thus here appear on an equal footing, and the conditions for violation of the CHSH inequality become identical for both. Indeed, only by giving up superdeterminism and allowing for freedom of choice for the settings do we see the asymmetry between settings and outcomes arise. The assumption of freedom of choice of the settings ensures that, in order to get a violation of the Bell inequality, the message $\mathcal{X}$ *must* contain information about the setting, either implicit or explicit (although note that SI can be satisfied). It is, however, not necessary that it carries information about the outcome.

Now we study explicit examples of non-local realistic models that violate the CHSH inequality. In all of them the non-local information $\mathcal{X}$ is information about the distant setting.

Consider the model of Toner and Bacon [6]. One of the parties sends the bit $\mathcal{X} = \pm 1$, which is given by $\mathcal{X} = \text{sgn}(\vec{b} \cdot \vec{\lambda}_1)\,\text{sgn}(\vec{b} \cdot \vec{\lambda}_2)$, where $\vec{b}$ is a unit Bloch vector corresponding to Bob's setting, and $\vec{\lambda}_1$ and $\vec{\lambda}_2$ are also unit vectors, which play the role of hidden variables. The communication in this model can be compressed to $C \approx 0.85$ bits [6]. Exactly the same value is obtained for the mutual (Shannon) information between the bit sent and the setting, $I(\mathcal{X} : \vec{b}) = C$.

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[9] A random binary hidden variable $\lambda = 0, 1$ is distributed to Alice and Bob. Bob's result for setting b is defined as $B = \lambda \oplus b$. Next, he communicates his outcome, $\mathcal{X} = B$. The result of Alice is given by $A = a(\mathcal{X} \oplus \lambda) \oplus \mathcal{X}$. Thus $J(\lambda, \mathcal{X} \to B) = 1$ and $J(\lambda \to B) = \frac{1}{2}$. Clearly, $A \oplus B = ab$, and the CHSH inequality is maximally violated. However, there is *intrinsic setting information* in this model because Alice can read the setting of Bob from the data available to her, $b = \mathcal{X} \oplus \lambda$; and thus $J(\lambda, \mathcal{X} \to b) = 1$ as well. This is what allows violation of the CHSH inequality. Note that this model is deterministic and thus obeys OI (and violates SI), despite the fact that it is the outcome B that is being communicated.

Table 1. The possibility of violation of the CHSH inequality in a local realistic model augmented by the communication of $\mathcal{X}$ from Bob to Alice. $J(\lambda, \mathcal{X} \to b)$ and $J(\lambda, \mathcal{X} \to B)$ are the 'guessed "information"' (GI) by Alice, where λ denotes the hidden variables, and b and B are Bob's setting and outcome, respectively. $\Delta_\lambda(\mathcal{X} \to b)$ and $\Delta_\lambda(\mathcal{X} \to B)$ denote the 'transmitted "information"' (TI) to Alice about Bob's setting and outcome, respectively, which is communicated via $\mathcal{X}$. (See the main text for their definitions.) 'No' in the right column means that the corresponding condition has to be violated to allow violation of the CHSH inequality. 'Yes' means that there are models that satisfy the condition and violate the CHSH inequality. The starred 'Yes*' in rows 4 and 6 indicate that for a violation either one of these conditions can hold, but not both. Similar is true for the doubly starred 'Yes**' in rows 7 and 8, where for completeness we have included the previously known results in terms of SI and OI.

Condition	Violation of CHSH possible?
$J(\lambda, \mathcal{X} \to b) = \frac{1}{2}$	No
$J(\lambda, \mathcal{X} \to B) = \frac{1}{2}$	No
$J(\lambda \to b) = \frac{1}{2}$	Yes ('freedom')
$J(\lambda \to B) = \frac{1}{2}$	Yes*
$\Delta_\lambda(\mathcal{X} \to b) = 0$	No
$\Delta_\lambda(\mathcal{X} \to B) = 0$	Yes*
SI: $P(A|a, b, \lambda) = P(A|a, \lambda)$	Yes**
OI: $P(A|a, b, B, \lambda) = P(A|a, b, \lambda)$	Yes**

The model of [6] perfectly simulates all possible measurement results obtained on the singlet state. If one aims at simulation of the maximal violation of the CHSH inequality (with four fixed settings) allowed by quantum mechanics, then less communication is needed, as shown in the following toy model with $\Delta_\lambda(\mathcal{X} \to B) = 0$. A binary random variable $\lambda = 0, 1$ is distributed from the source to Alice and Bob. His outcome for any choice of the setting is defined as $B = \lambda$, implying $J(\lambda \to B) = 1$ and thus $J(\lambda, \mathcal{X} \to B) = 1$ and $\Delta_\lambda(\mathcal{X} \to B) = 0$. If Bob's setting is $b = 0$, he sends always $\mathcal{X} = 0$; if his setting is $b = 1$, he sends $\mathcal{X} = 1$ with probability $p = \sqrt{2} - 1 \approx 0.414$ and $\mathcal{X} = 0$ otherwise. The outcome of Alice is given by $A = a\mathcal{X} \oplus \lambda$. In this model, it is the information about the setting of Bob that is communicated: the information content of $\mathcal{X}$ is 0.736 bits and this is the mutual information $I(\mathcal{X} : \vec{b})$ between $\mathcal{X}$ and the setting of Bob. Note that classical players exchanging this amount of information achieve efficiency of quantum solutions to communication complexity problems and games based on the CHSH inequality [5].

In the Leggett-type [7] non-local model of [14], a real unit vector, i.e. an infinite number of bits, parameterizing the setting is being sent from one party to another; thus $\Delta_\lambda(\mathcal{X} \to b) > 0$. Note that this model violates SI but obeys OI because it is deterministic [2].

Our last example is Bohm's theory [8]. Although here there is no explicit communication process, the specific dynamics of this theory allow setting information to be non-locally available. The information about the setting of the apparatus in one laboratory enters the formula for the velocity of the particle in the other laboratory. The analysis of the double

Stern-Gerlach experiment shows that the velocity of one of the particles is given by the equation [15] $v_1 = c_1 \tanh(c_2 \kappa)$, where κ is a parameter that describes the ratio between the magnetic field strengths at the two distant laboratories. The constants c_1 and c_2 do not depend on κ. Therefore, the local measurement outcome and the knowledge about the velocity *would* allow us to infer the distant setting. Since tanh is an injective function, to determine v_1, all the bits defining κ have to be known to the mechanism that generates this velocity.

This last example shows that there need not be an actual communication process and our results are valid outside the one-way communication paradigm. Indeed, it is irrelevant for our results how Alice obtained the information $\mathcal{X}$; one can think of it as extra information about Bob's situation and which is *somehow* available to Alice.

In conclusion, this work provides the general conditions that every non-local hidden-variable theory has to satisfy in order to allow for violation of the CHSH inequality. For there to be such a violation it must be the case that information about *both* the outcome and the setting at one laboratory is *available* at the distant one, despite the fact that there is *no need* for both non-local setting and outcome dependence in the conditional (hidden) probabilities. The role of the setting is shown to be fundamentally different from that of the outcome, and this asymmetry is shown to be due to the assumption of the experimenter's free setting choice. Because of this freedom, the only way to learn a distant setting is to have non-local information transferral. By contrast, it is possible that the distant outcome can also be learnt from the shared hidden variables, without any such non-local information transferral. The necessity that—within hidden variable models and freedom of choice—information about freely chosen distant settings has to be available in a space-like separated way seriously questions the possibility of Lorentz-invariant completion of quantum mechanics. This remark applies to both deterministic [16] and stochastic models and clearly goes beyond what can be concluded on the basis of an analysis using only the conditions SI and OI.

Acknowledgments

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