

Quantum networks reveal quantum nonlocality

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The results of local measurements on some composite quantum systems cannot be reproduced classically. This impossibility, known as quantum nonlocality, represents a milestone in the foundations of quantum theory. Quantum nonlocality is also a valuable resource for information processing tasks, *e.g.* quantum communication, quantum key distribution, quantum state estimation, or randomness extraction. Still, deciding if a quantum state is nonlocal remains a challenging problem. Here we introduce a novel approach to this question: we study the nonlocal properties of quantum states when distributed and measured in networks. Using our framework, we show how any one-way entanglement distillable state leads to nonlocal correlations. Then, we prove that nonlocality is a non-additive resource, which can be activated. There exist states, local at the single-copy level, that become nonlocal when taking several copies of it. Our results imply that the nonlocality of quantum states strongly depends on the measurement context.

Nonlocality is a property of the outcome distributions resulting from local measurements on composite physical systems. Consider a system shared by N parties, which perform m spacelike separated measurements with r possible results, on their respective subsystems. Denote by $x_i = 1, \dots, m$ the measurement chosen by party i and by $a_i = 1, \dots, r$ the corresponding outcome. The obtained joint probability distribution $P(a_1, \dots, a_N | x_1, \dots, x_N)$ is local whenever it can be explained as the result of classically correlated data, represented by λ , following a distribution $p(\lambda)$, *i.e.*

$$P_{\text{local}}(a_1, \dots, a_N | x_1, \dots, x_N) = \sum_{\lambda} p(\lambda) P(a_1 | x_1, \lambda) \dots P(a_N | x_N, \lambda). \quad (1)$$

Every local distribution (1) satisfies linear constraints known as Bell inequalities. The violation of a Bell inequality is then a signature of nonlocality: distributions not fitting the description (1) are called nonlocal.

Remarkably, local measurements on some quantum states lead to nonlocal correlations [1]. This can be observed in the original Bell test scenario [1], where the measurements are performed on a single copy of some quantum state ρ . In other words, there exist quantum distributions of outcomes

$$P_Q(a_1, \dots, a_N | x_1, \dots, x_N) = \text{Tr}(\rho M_{a_1}^{x_1} \otimes \dots \otimes M_{a_N}^{x_N}), \quad (2)$$

where $M_{a_i}^{x_i}$ are the positive operators defining the local measurements by party i , $\sum_{a_i} M_{a_i}^{x_i} = 1$, $\forall x_i$, which cannot be represented by any local model (1). The quantum state ρ is then called nonlocal. Contrary, if the results of all local measurements on ρ can be written as in (1), it is local at the single-copy level.

A stronger version of nonlocality is possible in a multipartite scenario. Consider the case in which only a subset of the N parties share nonlocal correlations. Although these correlations are nonlocal, they are not genuine N -partite nonlocal. A quantum state is said to be genuine

N -partite nonlocal only when there exist measurements on it establishing nonlocal correlations among all the N parties (see also Appendix A).

Deciding whether a quantum state is nonlocal represents not only a fundamental question; it also has important practical applications. The success of information-processing applications as quantum communication complexity [2], no-signalling [3] and device-independent [4] quantum key distribution, device-independent quantum state estimation [5, 6], or randomness extraction [7, 8], crucially relies on the existence of nonlocal correlations. Unfortunately, identifying the nonlocal properties of even the simplest families of entangled quantum state remains an extremely difficult problem [9–12].

Here we introduce a new framework to the study of quantum nonlocality. Given an N -partite quantum state ρ , the main idea is to create a network of $n \geq N$ parties and to distribute among them L copies of ρ , according to a given spatial configuration (see Figure 1). If nonlocal correlations are observed in the network, they can only come from the quantum state ρ and, therefore, it is called a nonlocal resource. Within this new scenario, we prove that all one-way entanglement distillable states are nonlocal resources. Then, we provide examples of activation of quantum nonlocality, namely quantum states that are local at the single-copy level but which become nonlocal in a network scenario or simply by taking several copies of it. We show that similar activation phenomena occur for genuine n -partite nonlocality.

There exist two main reasons for our network scenario to offer new possibilities in the study of quantum nonlocality. First, it enlarges the standard scenario in which a single-copy of an N -partite state is given to N observers. Second, the network scenario allows considering protocols in which a subset of the parties projects the remaining ones into a nonlocal quantum state. Indeed, post-selection is not a valid operation in standard bipartite Bell tests, as it is associated with sending information between the parties. Nevertheless, it is now allowed be-

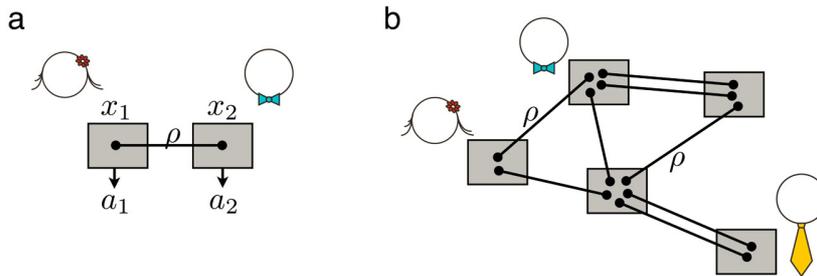


FIG. 1: **Nonlocality in the network scenario.** **a.** The standard scenario for the study of quantum nonlocality consists of N parties sharing one copy of an N -party quantum state ρ , to which they apply m different measurements of r possible results. The measurement choices and corresponding results by the two parties are labeled by $x_1, \dots, x_N = 1, \dots, m$ and $a_1, \dots, a_N = 1, \dots, r$. By repeating this process, the parties can estimate the joint probability distribution $P(a_1, \dots, a_N | x_1, \dots, x_N)$ describing the measurement statistics and test its nonlocality. **b.** We introduce a new scenario, where many copies of the quantum state ρ are distributed among n parties according to a network configuration. The parties measure their respective subsystems and check if the obtained multipartite probability distribution is local, *i.e.* if it can be written as in (1). As shown in the main text, there are quantum states that are local according to scenario **a**, but provide nonlocal correlations in scenario **b**.

cause it is performed on the results of measurements that are spacelike separated from the measurements actually used in the Bell test. (Spacelike separation here plays the role of time-ordering in the *hidden nonlocality* framework [13, 14]). These ideas are behind the two main technical observations used next. Consider an N -partite quantum state ρ .

Observation 1. If there exist local measurements by k parties such that, for one of the measurement outcomes, the resulting state among the remaining $N - k$ parties is nonlocal, then the initial state ρ is necessarily nonlocal.

This fact was first used in Ref. [15] to prove that all multipartite entangled pure states are nonlocal. In Appendix B we provide a simple proof of Observation 1.

Observation 2. If, for every bipartition of the parties, there exist local measurements on $N - 2$ parties that, for every outcome, create a maximally-entangled state between parties belonging to the different partitions, then the initial state ρ is genuine N -partite nonlocal.

This was proven in Ref. [16] and is actually stronger as it also implies that the state ρ contains only genuine N -partite nonlocal correlations.

Our techniques are fully general and apply to any state. However, in what follows, we frequently illustrate their usefulness in networks composed of isotropic states in $d \times d$ systems; namely mixtures of maximally-entangled states, $|\Phi\rangle$, and white noise, weighted by the noise parameter p ,

$$\rho_I(p) = p |\Phi\rangle\langle\Phi| + (1-p) \frac{1}{d^2}. \quad (3)$$

The known nonlocal properties of these states are summarized in Figure 2.

We start by showing that any one-way entanglement distillable states form a quantum network that displays nonlocal correlations. The required network consists of three parties in a Λ configuration (see Figure 3): one of the parties, say Alice, shares L copies of ρ with Bob,

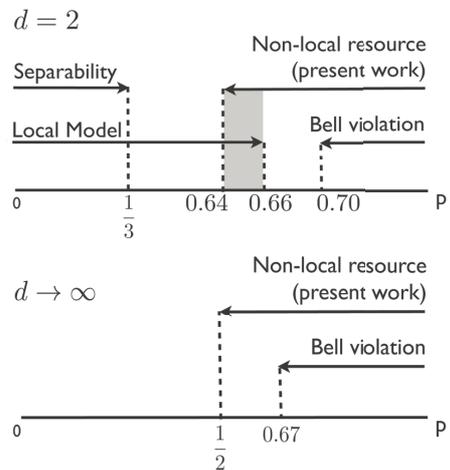


FIG. 2: **Nonlocality of isotropic states.** Isotropic states are mixtures of a maximally-entangled state and white noise, see (3). In the qubit case, $d = 2$, these states violate a Bell inequality for $p \gtrsim 0.705$ [22]. A local model for any experiment involving von Neumann measurements on a single copy of these states is possible for $p \lesssim 0.66$ [10]. This limit also holds if general two-outcome measurements are considered (see Appendix D). In the case of general measurements, the existence of a local model has been proven for $p \leq 5/12$ [11]. Here we show, using two-outcome measurements, that two-qubit isotropic states are nonlocal in a network scenario for $p \gtrsim 0.64$. In the limit of very large dimension, $d \rightarrow \infty$, isotropic states violate the Collins-Gisin-Linden-Massar-Popescu (CGLMP) Bell inequality [23] for $d \gtrsim 0.67$. We show that these states are nonlocal resources in our network scenario for $p > 1/2$.

and L different copies with Charlie. If ρ is one-way entanglement distillable, there exists a measurement result by Alice which projects Bob and Charlie into a state that can be made, by increasing L , arbitrarily close to a maximally-entangled state. As this state is nonlocal,

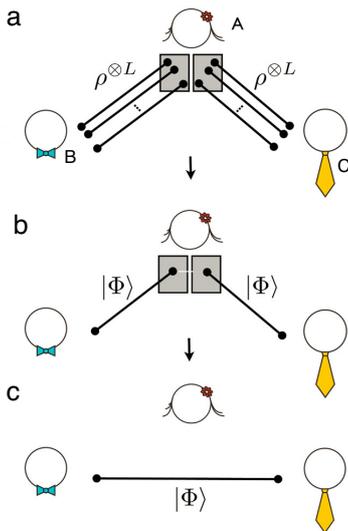


FIG. 3: **Any one-way entanglement distillable state is a nonlocal resource.** **a.** Consider a network consisting of three distant parties Alice, Bob, and Charlie. Alice shares L copies of an entangled state with Bob, ρ_{AB} , and L copies of the same state with Charlie, ρ_{AC} . **b.** Assume this state has non-zero one-way distillable entanglement. Then, there exists a projection on Alice's part of $\rho_{AB}^{\otimes L}$ such that the resulting state between Alice and Bob, σ_{AB} , tends to a maximally-entangled state, in the limit of an infinite number of copies, *i.e.* $\langle \Phi | \sigma_{AB} | \Phi \rangle \rightarrow 1$ when $L \rightarrow \infty$. The same projection is applied by Alice to her part of $\rho_{AC}^{\otimes L}$. **c.** Alice projects now her subsystems into a maximally-entangled state. This, in turn, projects Bob and Charlie into a state which is arbitrarily close to a maximally-entangled state. Clearly, all the previous projections by Alice can be seen as a one-shot measurement that, with non-zero probability, leaves Bob and Charlie in a state arbitrarily close to a maximally-entangled state. Since this state is nonlocal, we conclude that the network state is also nonlocal (Observation 1). In the case of isotropic states (3), this procedure allows the detection of nonlocality for $p > 1/2$ when $d \rightarrow \infty$. This value is obtained by computing the hashing bound, which is a lower bound on the one-way distillable entanglement [24], of these states.

the network state must be nonlocal by virtue of Observation 1. As an illustration, consider a network made of isotropic states (3). In the limit of large dimension, non-local correlations are observed whenever $p > 1/2$, which is far beyond the known region of Bell violation for the single-copy ρ_I (see Figure 2).

We now move to the announced examples of activation of quantum nonlocality. The first example is concerned with genuine multipartite nonlocality and is remarkably simple. Consider a bipartite maximally-entangled state $|\Phi\rangle$, which clearly is not genuine tripartite nonlocal. However, two copies of $|\Phi\rangle$ disposed in a Λ configuration, as above, compose a network with genuine tripartite nonlocality. The network state reads $|\Phi\rangle_{AB} \otimes |\Phi\rangle_{A'C}$, which is tripartite nonlocal according to Observation 2 (actually, it is fully tripartite nonlocal [16]). This example can

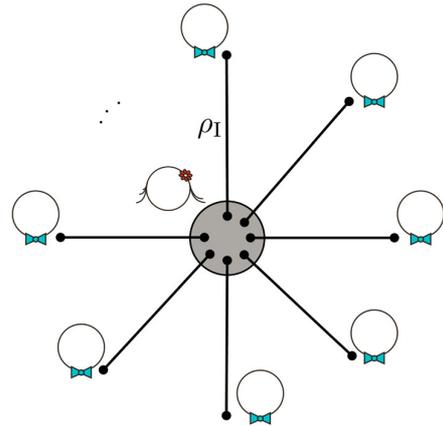


FIG. 4: **Activation of nonlocality.** Consider the network scenario in which $N + 1$ parties share two-qubit isotropic states ρ_I in a star-like configuration. If the central node, Alice, projects her qubits into the state $|GHZ\rangle = (|0\dots 0\rangle + |1\dots 1\rangle)/\sqrt{2}$, the remaining N parties are left with a state that violates a Bell inequality for $p > p_N = (2/\pi)2^{1/N}$ [17]. It then follows from Observation 1 that the network state is nonlocal and ρ_I is a nonlocal resource in this region. For $N \geq 7$, this bound is smaller than the noise corresponding to the best known Bell violation [22], *i.e.* $p_7 < 0.705$. More interestingly, if $N \geq 21$ one has $p_N < 0.66$. This is precisely the noise value for which the existence of a local model for von Neumann measurements on ρ_I has been proven [10]. As the local model in Ref. [10] can easily be extended to general two-outcome measurements (see Appendix D) and the previous Bell violation only involved two-outcome measurements, we conclude that the nonlocality of ρ_I is activated by the network configuration. That is, nonlocal correlations are observed in networks consisting of isotropic states that are local at the single-copy level.

be used to construct a more standard example of activation, where the tripartite quantum nonlocality of a state is activated by the state itself. The state reads

$$\sigma = \frac{1}{2} (|\psi_1\rangle\langle\psi_1|_{ABC} \otimes |000\rangle\langle 000|_{A_f B_f C_f} + |\psi_2\rangle\langle\psi_2|_{ABC} \otimes |111\rangle\langle 111|_{A_f B_f C_f}), \quad (4)$$

where $|\psi_1\rangle_{ABC} = |\Phi\rangle_{AB} \otimes |\phi\rangle_C$ and $|\psi_2\rangle_{ABC} = |\Phi\rangle_{AC} \otimes |\phi\rangle_B$, being $|\phi\rangle$ some arbitrary state. Now, although σ does not contain any genuine tripartite nonlocality, $\sigma^{\otimes L}$ becomes genuine tripartite nonlocal for large L (see Appendix C). Such results can be generalized to an arbitrary number of parties: a star-shape n -party network, in which the central node is connected to the remaining nodes by one copy of a state $|\Phi\rangle$, is genuine n -partite nonlocal.

Moving to standard nonlocality, our example of activation uses isotropic two-qubit states (3) disposed again in a star-shape network, as depicted in Fig. 4. In the noise region $p \gtrsim 0.64$, there exists a local measurement result in the central node that leaves the remaining subsystems in a nonlocal state [17]. Then, Observation 1

guarantees that the corresponding isotropic states are nonlocal resources. But, when $p \lesssim 0.66$, these states are known to be local at the single-copy level for von Neumann measurements [10] (and, consequently, for general dichotomic measurements, see Appendix D). In the overlap of these regions, nonlocality is activated: local (single-copy) isotropic states form a nonlocal quantum network. The minimal size of the nonlocal network required to enter the single-copy locality region is 21 parties. As in the previous example (4), classically correlated states can be added to construct an example of activation in which a 21-particle local state τ becomes nonlocal by taking a sufficiently large number of copies of it, $\tau^{\otimes L}$ (see Appendix C).

To conclude, here we show that a much better use of the nonlocal potential of quantum states is achieved simply by distributing them in networks. Our work opens new perspectives in the study of quantum nonlocality. For instance, it would be interesting to analyze if, for any entangled state, there exists a network displaying nonlo-

cal correlations. In fact, despite considerable efforts after Bell's seminal work, we are still far from understanding the exact relation between entanglement and nonlocality. Our results show that this relation is much subtler than initially expected: the nonlocal character of a quantum state strongly depends on its measurement context, namely on the network configuration.

Acknowledgments

We thank A. Winter and A. Salles for interesting comments. This work was supported by the European Q-Essence project and ERC Starting grant PERCENT, the Spanish MEC FIS2007-60182 and Consolider-Ingenio QOIT projects, Generalitat de Catalunya, Caixa Manresa, National Research Foundation and the Ministry of Education of Singapore.

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I. APPENDICES

A. Definition of genuine multipartite nonlocality.

In this section we provide a formal definition of genuine multipartite nonlocality, a stronger version of nonlocality that can be defined in the multipartite scenario [16, 18–21]. Consider, for instance, a situation in which the correlations observed among N parties can be written as

$$P_{\text{L:NL}}(a_1, \dots, a_N | x_1, \dots, x_N) = \sum_{\lambda} p(\lambda) P(a_1, \dots, a_k | x_1, \dots, x_k, \lambda) \times P(a_{k+1}, \dots, a_N | x_{k+1}, \dots, x_N, \lambda). \quad (5)$$

That is, the distribution can be simulated by a hybrid local/nonlocal model in which nonlocal correlations are given to the first k parties and to the $N - k$ remaining ones, but these two groups are correlated only through the classical random variable λ . Despite the fact that hybrid distributions (5) can be nonlocal, in the sense of violating (1), they are not genuinely multipartite nonlocal, *i.e.*, the nonlocal correlations are not shared by all the parties in the system. This observation naturally leads to the concept of genuine N -partite nonlocality.

A probability distribution is said to be genuine N -partite nonlocal, or contain genuine N -partite correlations, whenever it cannot be reproduced by the combination of any hybrid models (5), that is,

$$P_{\text{MNL}}(a_1, \dots, a_N | x_1, \dots, x_N) \neq \sum_{\lambda} p(\lambda) P_{\text{L:NL}}^{\lambda}. \quad (6)$$

Here, the terms $P_{\text{L:NL}}^{\lambda}$ are such that there exists a splitting of the N parties into two groups, which may depend on λ , allowing a decomposition like in (5). Consequently, an N -partite quantum state is said to be genuine N -partite nonlocal whenever there exist local measurement by the parties leading to genuine N -partite nonlocal correlations.

It is important to mention here that, throughout this work, we always assume the validity of the no-signalling principle. Thus, all the terms $P_{\text{L:NL}}^{\lambda}$ must be compatible with such principle (see also [21]). This does not coincide with the original definition of genuine N -partite nonlocality given in Refs. [18–20] but has a natural operational meaning: the parties are assumed not to be able to transmit information instantaneously even if they have access to the variable λ .

Similar to standard nonlocality, genuine multipartite nonlocality is usually detected by the violation of some linear inequalities, known as Svetlichny inequalities, that are satisfied by any hybrid local/nonlocal distributions [18–20].

B. Constructing Bell inequality for N -partite systems

In this section we prove Observation 1. The goal, then, is to show that if an N -party state ρ is such that there exist local projections by k parties mapping the remaining $N - k$ parties into a nonlocal state, ρ_{N-k} , then the initial state ρ is also nonlocal. The proof is based on the fact that it is always possible to construct a Bell inequality violated by the state ρ from a Bell inequality violated by the post-measurement nonlocal state ρ_{N-k} .

For simplicity in the notation, we consider the case in which the local projections are performed by the first k parties. Take any Bell inequality violated by the nonlocal state ρ_{N-k} ,

$$\mathcal{I}_{N-k} = \sum_{\mathbf{a}, \mathbf{x}} c_{\mathbf{a}, \mathbf{x}} P(\mathbf{a} | \mathbf{x}) \leq K, \quad (7)$$

where $\mathbf{a} = a_{k+1} \dots a_N$ represent the outcomes of the measurements $\mathbf{x} = x_{k+1} \dots x_N$ performed by the parties $k + 1, \dots, N$, and $c_{\mathbf{a}, \mathbf{x}}$ and K are the weights and local bound defining the inequality. Since the state ρ_{N-k} is obtained for a particular outcome $\mathbf{b}' = b'_1 \dots b'_k$ of local measurements $\mathbf{y}' = y'_1 \dots y'_k$ at parties $1, \dots, k$ applied to ρ , we can use (7) to write a generalized Bell inequality [15] violated by ρ ,

$$\mathcal{I}_N = \sum_{\mathbf{a}, \mathbf{x}} c_{\mathbf{a}, \mathbf{x}} P(\mathbf{a} | \mathbf{x}, \mathbf{b}', \mathbf{y}') \leq K. \quad (8)$$

Generalized Bell inequalities differentiate from standard ones in the sense that they consider distributions conditioned on particular measurement outcomes, *i.e.* they assume post-selection of events.

In general, it is not possible to transform a generalized Bell inequality into a standard Bell inequality. However, this turns out to be possible in our case because the local measurements and results corresponding to the post-selection, \mathbf{y}' and \mathbf{b}' , define events that are space-like separated from the measurements appearing in the Bell inequality (7), \mathbf{x} . The no-signalling condition then guarantees that the event $(\mathbf{b}', \mathbf{y}')$ can be written independently from measurements \mathbf{x}

$$P(\mathbf{a}, \mathbf{b}' | \mathbf{x}, \mathbf{y}') = P(\mathbf{b}' | \mathbf{y}') P(\mathbf{a} | \mathbf{x}, \mathbf{b}', \mathbf{y}'). \quad (9)$$

Using this, it directly follows from inequality (8) that the N -partite state ρ must violate

$$\mathcal{I}_N = \sum_{\mathbf{a}, \mathbf{x}} c_{\mathbf{a}, \mathbf{x}} P(\mathbf{a}, \mathbf{b}' | \mathbf{x}, \mathbf{y}') \leq K P(\mathbf{b}' | \mathbf{y}'), \quad (10)$$

which is a standard Bell inequality satisfied by all local models (1).

C. Activation of nonlocality in the many-copy scenario.

The purpose of this section is to show how the examples of activation described in the main text, which are

based on network configurations, can be mapped into more standard examples of activation, in which the non-local properties of a quantum state change by taking copies of it. The main idea is to provide all the parties with classically correlated *flags* which allow them to reconstruct the activation network, with a probability that can be made arbitrarily close to one by increasing the number of copies. We illustrate this procedure in the simplest example of activation of genuine multipartite nonlocality, but the same construction can be easily applied to the example of activation of standard nonlocality.

Consider the tripartite state

$$\sigma = \frac{1}{2}(|\psi_1\rangle\langle\psi_1|_{ABC} \otimes |000\rangle\langle 000|_{A_f B_f C_f} + |\psi_2\rangle\langle\psi_2|_{ABC} \otimes |111\rangle\langle 111|_{A_f B_f C_f}), \quad (11)$$

introduced in the main text, where $|\psi_1\rangle_{ABC} = |\Phi\rangle_{AB} \otimes |\phi\rangle_C$ and $|\psi_2\rangle_{ABC} = |\Phi\rangle_{AC} \otimes |\phi\rangle_B$, being $|\phi\rangle$ some arbitrary state. Clearly this state has only bipartite nonlocal correlations. We show in what follows that $\sigma^{\otimes L}$, for sufficiently large L , is genuine tripartite nonlocal. As said, in order to do that it is convenient to interpret the qubits in systems A_f , B_f and C_f as flags. Moreover, it is important to recall that two bipartite maximally-entangled states in a Λ configuration, $|\Psi\rangle = |\Phi\rangle_{AB}|\Phi\rangle_{A'C}$, define a genuine tripartite nonlocal state according to Observation 2. This means that there exist local measurements by the parties, M_x^a , M_y^b and M_z^c , such that the corresponding correlations, described by

$$P_\Psi(abc|xyz) = \text{Tr}(|\Psi\rangle\langle\Psi|M_x^a \otimes M_y^b \otimes M_z^c), \quad (12)$$

are genuine multipartite nonlocal, *i.e.* violate a Svetlichny-like Bell inequality [18–20] for the detection of genuine multipartite nonlocality in the no-signalling framework (see discussion at the end of Appendix A).

Given the L copies of σ , the parties apply the following measurement strategies. Alice measures the flag A_f of her first copy of σ in the $\{|0\rangle, |1\rangle\}$ basis. Without loss of generality, assume her result is equal to 0. Then, she knows she shares a maximally-entangled state with Bob in the AB system. She keeps measuring the remaining flags until she gets outcome 1. Then, she also shares a maximally-entangled state with Charlie in the AC systems. That is, Alice has effectively prepared two singlets in a Λ configuration. She now applies the measurement M_x^a , given in (12), to the two particles associated to the two flags where she got first results 0 and 1. Bob and Charlie apply a similar strategy: Bob (Charlie) measures the flags B_f (C_f) in the computational basis until he finds result 0 (1). Then he applies the measurement M_y^b (M_z^c) to the corresponding B (C) particle. Although presented in a sequential way for clarity reasons, this process defines in fact one-shot local measurements on each party. Clearly, the obtained correlations among the parties are the same as in (12) and, thus, are genuine tripartite nonlocal.

Of course, with probability $p_{\text{eq}} = 1/2^{L-1}$ all the flags give the same result. As it becomes clear in the next lines, these instances can be ignored, but we discuss them here for the sake of completeness. Assume that all the flags give 0 (1). Then, the parties can apply, for instance, the measurement strategy on L maximally-entangled states between Alice and Bob (Charlie) which optimally approximates $P_\Psi(abc|xyz)$. The obtained correlations are denoted by $P_\Psi^B(abc|xyz)$ ($P_\Psi^C(abc|xyz)$), although their explicit form is irrelevant for our considerations.

Putting all these possibilities together, the resulting probability distribution among the three parties is equal to

$$\frac{p_{\text{eq}}}{2} \tilde{P}_\Psi^B(abc|xyz) + \frac{p_{\text{eq}}}{2} \tilde{P}_\Psi^C(abc|xyz) + (1-p_{\text{eq}})P_\Psi(abc|xyz). \quad (13)$$

This can be made arbitrarily close to the tripartite nonlocal distribution $P_\Psi(abc|xyz)$, as p_{eq} tends to zero exponentially with the number of copies L . Therefore, there always exists a finite value of L (that might not be large), such that genuine tripartite nonlocal correlations can be obtained from $\sigma^{\otimes L}$, although σ was not genuine tripartite nonlocal.

The example of activation of standard nonlocality can be obtained following a similar procedure. As above, we combine classically correlated flags with the activation result in the network scenario to build an $N + 1$ -partite quantum state τ which is local, but such that $\tau^{\otimes L}$ is nonlocal for large L . The state τ reads

$$\begin{aligned} \tau_{AB_1 B_2 \dots B_N} &= \\ &= \frac{1}{\sqrt{N}} \sum_{i=1}^N \rho_{AB_i} \otimes |i_{A_f} i_{B_{1,f}} \dots i_{B_{N,f}}\rangle\langle i_{A_f} i_{B_{1,f}} \dots i_{B_{N,f}}|, \end{aligned} \quad (14)$$

where

$$\rho_{AB_i} = \rho_1^{AB_i} \bigotimes_{j=1, j \neq i}^N \gamma_j^{B_j}. \quad (15)$$

represents the product of a two-qubit isotropic state $\rho_1^{AB_i}$, shared by parties A and B_i , with an arbitrary state $\gamma_j^{B_j}$ for the remaining ones. The states $|i_{A_f}\rangle$ and $|i_{B_{j,f}}\rangle$ provide the correlated flags among the parties. Using a similar measurement scheme as above, it is easy to prove that by increasing the number of copies L , $\tau^{\otimes L}$ can be deterministically transformed into a state arbitrarily close to the nonlocal star-configuration state of Fig.4.

D. Any two-outcome measurements can be simulated by von Neumann measurements.

Here we show that projective measurements are enough to simulate any outcome distribution obtained by dichotomic (*i.e.* two-outcome) general measurements.

Consider a dichotomic measurement described by elements M_0 and M_1 , which are positive operators such

that $M_0 + M_1 = 1$. Then, their spectral decompositions can be expressed in the same basis: $M_0 = \sum_i \lambda_i |\varphi_i\rangle\langle\varphi_i|$ and $M_1 = \sum_i (1 - \lambda_i) |\varphi_i\rangle\langle\varphi_i|$, with $0 \leq \lambda_i \leq 1$. Consequently, the results of this general measurement can be simulated by a protocol consisting of the following steps: (i) the von Neumann measurement defined by projectors $\{|\varphi_i\rangle\langle\varphi_i|\}$ is applied and (ii) depending on the observed outcome, i , the observer outputs 0 with probability λ_i and 1 with probability $1 - \lambda_i$.

This simple observation implies that a local model for projective measurements on a quantum state also applies to general dichotomic measurements. This turns out to be particularly relevant for our example of activation of nonlocality. Recall that this example is based on the fact that the star-shape network made of isotropic states, see Fig. 4, is nonlocal for a noise threshold $p > 0.64$. It is however crucial that dichotomic measurements are sufficient to reveal the nonlocality of the network [17]. While it is unknown whether there exist isotropic states with $p > 0.64$ that are local under general measurements, as the best known model works for $p \leq 5/12$ [11], they are certainly local for two-outcome measurements when $p < 0.66$. This easily follows from the previous result and the existence of a local model for projective measurements when $p < 0.66$ [10] (see Fig.2). We then conclude that isotropic states have their non-locality activated: there exist states local under dichotomic measurements at the single-copy level, but which have their nonlocality revealed by dichotomic measurements in the network scenario.